GCE

Further Mathematics A

Y543/01: Mechanics

A Level

Mark Scheme for June 2022

Oxford Cambridge and RSA Examinations

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Text Instructions

1. Annotations and abbreviations

Annotation in RM assessor	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	**Meaning**
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value **is given** in the paper only accept an answer correct to at least as many significant figures as the given value.

- When a value **is not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for *g* should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	(a)		$10000 = D \times 5 \Rightarrow D = 2000$	M1	3.4	Using "$P = Fv$" to find the 'driving force'	
			$2000 - 250 = 1200a$	M1	1.1	Using NII with two force terms in opposite directions to derive an equation in a	
			$a = 1.458...$ so acceleration is awrt 1.46 ms^{-2}	A1 [3]	1.1		
	(b)		At max speed, $250 = 20000/v$	M1	3.4	Using NII with $a = 0$ oe and "$P = Fv$" with maximum power	
			$\Rightarrow v = 80$ so max speed is 80 ms^{-1}	A1 [2]	1.1		
	(c)		$20000/v = 250 + 1200g\sin\theta$	M1	3.4	Using NII with $a = 0$ oe and three force terms and "$P = Fv$" with maximum power	$20000 = 838v$ Allow sign slip
			$v = 23.86...$ so max speed is awrt 23.9 ms^{-1}	A1 [2]	1.1		

Question			Answer	Marks	AO	Guidance	
2	(a)		Displacement = $(5 - -1)\mathbf{i} + (12 - 6)\mathbf{j}$ m	M1	1.1	Subtracting the position vectors of the points to find the displacement $6\mathbf{i} + 6\mathbf{j}$ m	
			$(7\mathbf{i} - 2\mathbf{j}).(6\mathbf{i} + 6\mathbf{j}) = 42 + -12 = 30$ so 30 J	A1 [2]	1.1		
	(b)		$P = \mathbf{F.v} = (7\mathbf{i} - 2\mathbf{j}).(-\mathbf{i} - 5\mathbf{j})$ $= -7 + 10 = 3$ so 3 W	M1 A1 [2]	1.1 1.1	Use of $P = \mathbf{F.v}$	
	(c)		$OB = \sqrt{(5^2 + 12^2)} = 13$ so extension is 5m	M1	3.1b	Calculating OB and hence finding the extension	
			$T = 24 \times (\text{"OB"}-8) / 8\ (= 15)$ or EPE $= 24 \times (\text{"OB"}-8)^2 / (2\times8)\ (= 37.5)$	M1	1.1		
			$T = 24 \times 5 / 8 = 15$ so **15 N** and EPE $= 24 \times 5^2 / (2\times8) = 37.5$ so **37.5 J**	A1 [3]	1.1		If M0M1A0 SC1 for T = 1.46N and EPE = 0.353J

Question			Answer	Marks	AO	Guidance	
3	(a)		$I = \int_0^2 \frac{1}{5-4e^{-t^2}} dt$	M1	1.1	Correct form of integral and correct limits.	
			= 0.81365... so impulse is awrt 0.814 Ns	A1 [2]	1.1	BC	Accept awrt 0.81 or 0.82 if correct integral shown
	(b)		Impulse = change in momentum so "0.81365..." = $\pm(6v - 6\times1.9)$	M1	1.1	Use of impulse-momentum principle	
			$v = 1.9 + 0.81365.../6 = 2.0356...$ so *P*'s speed is awrt 2.04 ms^{-1}	A1 [2]	1.1		Accept awrt 2.03 or 2.04
	(c)		WD = change in KE = ½×6("2.0356..."^{2} – 1.9^2) J	M1	1.1	Use of work-energy principle with values for v, m and u substituted in	
			= awrt 1.60 J	A1 [2]	1.1		Accept anything awrt 1.60 – 1.62

Question			Answer	Marks	AO	Guidance	
4	(a)		m_1 and m_2 have the **same dimensions** and NIII says that the **magnitude of the force that each applies to the other is the same**. So m_1 and m_2 must be **interchangeable** which means that the dependency on each must be the same and hence $\alpha = \beta$.	B1 **[1]**	2.4	Argument must be based on the idea that the dimensions/units of the quantities are equal and that they are interchangeable	Must reference at least 2 of the 3 bold statements
	(b)		$M^{-1}L^3T^{-2}$	B1 **[1]**	1.1	cao but condone change in order of M, L and/or T	Don't accept square brackets
	(c)		$[F] = MLT^{-2}$, $[m_i] = M$ and $[r] = L$ => M: $1 = -1 + 2\alpha$ L: $1 = 3 + \gamma$ => $\alpha = \beta = 1$ and $\gamma = -2$	M1 M1 A1 **[3]**	1.1 1.1 1.1	Setting up equation in α (and/or β) by considering mass dimension Ignore attempt to set up equation involving time dimension Setting up equation in γ by considering length dimension Ignore attempt to set up equation involving time dimension or $F = Gm_1m_2r^{-2}$	or $1 = -1 + \alpha + \beta$
	(d)		$Gm_1m_2r^{-2} = m_2r\omega^2$ $\omega = \sqrt{(6.67\times10^{-11} \times 5.97\times10^{24}/(3.84\times10^8)^3)}$ => $\omega = 2.65 \times10^{-6} = 2\pi / T$ => T = awrt 2370000 so period is 2370000/(60×60×24) = 27 days to the nearest day	M1* M1dep* A1 **[3]**	3.3 3.4 1.1	Using NII with **a** correct form for the centripetal acceleration Using $v = \frac{2\pi r}{T}$ or $\omega = \frac{2\pi}{T}$ with values substituted in	$T^2 = \dfrac{4\pi^2(3.84 \times 10^8)^3}{(6.67 \times 10^{-11})(5.97 \times 10^{24})}$

Question			Answer	Marks	AO	Guidance	
5	(a)		**DR** $\int_0^4 \frac{15}{\sqrt{x^2+9}}dx = 15\left[\ln\left(x+\sqrt{x^2+9}\right)\right]_0^4$ $=15(\ln 9 - \ln 3) = 15\ln 3$	**B1**	1.1	If this is not seen in **(a)** the mark can be awarded here if integral correctly evaluated in **(b)**.	
			$15\int_0^4 \frac{1}{2}\times 2x(x^2+9)^{-\frac{1}{2}}dx$ $=15\left[(x^2+9)^{\frac{1}{2}}\right]_0^4$	**M1**	1.1	Integrating. Ignore limits here.	Must be in the form $k\left[(x^2+9)^{\frac{1}{2}}\right]$
			$=15\left[(4^2+9)^{\frac{1}{2}}-(0^2+9)^{\frac{1}{2}}\right]$ $=15[5-3]=30$ $\Rightarrow \bar{x} = \frac{30}{15\ln 3} = \frac{2}{\ln 3}$	**A1** **[3]**	1.1	AG. Some intermediate working must be seen.	Award for fully complete proof only
	(b)		**DR** $\frac{1}{2}\int_0^4 \left(\frac{15}{\sqrt{x^2+9}}\right)^2 dx = \frac{225}{2}\int_0^4 \frac{1}{x^2+9}dx$	**M1**	1.1	Correct integral and correct limits	
			$=\frac{225}{2}\times\frac{1}{3}\left[tan^{-1}\frac{x}{3}\right]_0^4$	**M1**	1.1	Integrating into the form $k\left[tan^{-1}\frac{x}{3}\right]$	May have used substitution
			$=\frac{75}{2}\tan^{-1}\frac{4}{3}$ $\therefore \bar{y} = \frac{\frac{75}{2}\tan^{-1}\frac{4}{3}}{15\ln 3} = \text{awrt}2.11$	**A1** **[3]**	1.1		

	(c)		**DR** $x = 3 \Rightarrow y = \frac{15}{\sqrt{18}}$ oe $\tan\theta = \frac{3 - \frac{2}{\ln 3}}{"\frac{15}{\sqrt{18}}" - "2.1101\ldots"} = \frac{1.17952\ldots}{1.42538\ldots}$ $= 0.8275\ldots$ $\theta =$ awrt$39.6°$	**B1** **M1** **A1** **[3]**	3.4 2.1 2.2a	Finding the y coord of P. $= \frac{5}{\sqrt{2}} = \frac{5}{2}\sqrt{2}$ or awrt 3.54 $\tan\theta = \frac{\Delta x}{\Delta y}$ or $\frac{\Delta y}{\Delta x}$ 0.691rads	

Question			Answer	Marks	AO	Guidance	
6	(a)		$I = mv - mu = 2.5(18 - -5) = 57.5$	**B1** **[1]**	1.1		
	(b)		$F = ma$ $\therefore \frac{5v^2}{x} - 60v = 2.5v\frac{\mathrm{d}v}{\mathrm{d}x}$ $\therefore \frac{\mathrm{d}v}{\mathrm{d}x} = \frac{2v}{x} - 24$ so $a = 2$ and $b = -24$	**M1** **A1** **[2]**	3.3 2.2a	Using $F = ma$ to express a differential form of the acceleration in terms of the given forces with the signs correct	
	(c)		$\frac{dv}{dx} = \frac{2v}{x} - 24 \Rightarrow \frac{dv}{dx} - \frac{2}{x}v = -24$ $\Rightarrow \text{IF} = e^{\int -\frac{2}{x}dx} = e^{-2\ln x} = x^{-2}$ $x^{-2}\frac{\mathrm{d}v}{\mathrm{d}x} - 2x^{-3}v = \frac{\mathrm{d}}{\mathrm{d}x}\left(vx^{-2}\right) = -24x^{-2}$ $vx^{-2} = \int -24x^{-2}\mathrm{d}x = 24x^{-1} + c$ $x = 1, v = 18 \Rightarrow 18\times1^{-2} = 24\times1^{-1} + c \Rightarrow$ $c = -6 \Rightarrow v = 24x - 6x^2$	**B1FT** **M1*** **M1dep*** **A1** **[4]**	1.1 1.1 1.1 3.4	Correctly determining the integrating factor x^{-a} $a \neq 0$ Multiplying by the IF and writing LHS as an exact derivative. Integrating BS with a constant of integration Substituting initial conditions to find c	FT their value of a provided $a \neq 0$ IF must be in the form x^k or using a definite integral with suitable limits or using the limits correctly in a definite integral with suitable limits
	(d)		• As $t \to \infty$, $x \to 4$ OR P moves from its initial position to the point on the x-axis where $x = 4$ • P initially accelerates and then decelerates (to 0) • P reaches its max speed of 24ms^{-1} or reaches max speed when $t = \frac{1}{24}\ln 3$ (0.0458) or reaches max speed when x = 2	**B1** **B1** **B1** **[3]**	2.4 2.4 2.2a		

Question			Answer	Marks	AO	Guidance	
7	(a)		$\frac{1}{2} \times m \times 7^2 - \frac{1}{2} \times m \times 5.5^2$	**M1**	3.3	Change in KE considered in conservation of energy equation	Assuming the zero PE level is set at ground level. Any level can be used but if not ground level then it must be clearly defined.
			PE at general angle $\theta = mg \times (6.5 - 4.5\cos\theta)$	**M1**	3.1b	Expressing height difference in terms of θ which may be embedded Which must be correct	Here θ is angle with downward vertical. Allow sin/cos
			$3.5mg + \frac{1}{2} \times m \times 7^2 = mg(6.5 - 4.5\cos\theta) + \frac{1}{2} \times m \times v^2$	**M1**	3.3	Using their h in a conservation of energy equation involving KE and PE (could see 5.5 for v)	May see h = 4.45633 Allow sin/cos
			$-6 \times 9.8 + 7^2 - 5.5^2 = -9 \times 9.8\cos\theta$ => $\cos\theta = 89/196 = 0.45408...$	**A1**	1.1	Correct value for $\cos\theta$ (or θ)	$\theta = 63.0°$ or 1.099rads may see $\theta = 27.0°$ or 0.472
			NII => $T - mg\cos\theta = \frac{mv^2}{r}$	**M1**	3.3	NII with a correct form for centrepetal acceleration used	Allow sin/cos. Must see their values substituted in and a weight component
			=> $T = 90 \times 5.5^2/4.5 + 90 \times 9.8 \times 89/196 = 1005.5$ so magnitude of tension is 1005.5 N	**A1** **[6]**	1.1	AG. Intermediate step must be seen	
	(b)		$v_x = 5.5\cos\theta = 5.5 \times 0.45408 = 2.4974...$ and $v_y = 5.5\sin\theta = 5.5 \times 0.89096 = 4.90028$	**B1FT**	3.4	FT their $\cos\theta/\sin\theta$	
			$-(6.5 - 4.5\cos\theta) = -4.45663 = 4.90028t - \frac{1}{2} \times 9.8t^2$	**M1**	1.1	Using suvat to set up three term quadratic equation in t	Must use a component of 5.5 for u Height must be -ve if taken upwards as positive
			=> $t = 1.57685...$ or $-0.57679...$ but $t > 0$ => $t = 1.57685...$	**A1**	2.3	Could be BC. Must eliminate unfeasible solution	
			$OT = \sqrt{(4.5^2 - 3^2)} + 4.5 \times 0.89096 + 1.57685 \times 2.4974 = 3.354 + 4.009 + 3.938$	**M1**	3.4	Complete method for finding OT using their values.	
			= awrt 11.3 m	**A1** **[5]**	2.2a		

	(c)		$V_y = 4.90028 - 9.8\times1.57685 = -10.5528...$ => Init KE = $\frac{1}{2}\times90(2.4974^2 + (-10.5528)^2)$	**B1FT**	3.1b	FT their v_x, v_y or their initial energy from (a) 5291.98	Or use initial energy from (a)
			$\tan\phi = 10.5528/2.4974 = 4.2255...$ => $\phi = 76.685...°$ so $5291.98 + 90\times9.8\times0.3\sin76.685° = \lambda\times0.3^2/(2\times0.5)$	**M1**	3.1b	Conservation of energy with KE, GPE and EPE terms (two of these correct (can FT their KE)	Give M mark if only two components considered 58800 implies B1M1
			$90\times9.8\times0.3\sin76.685°$	**B1**	3.1b	GPE term correct	
			=> $\lambda = 61661$ so MoE is 61661 N	**A1cao** **[4]**	1.1	61740 comes from not considering angle	

Question			Answer	Marks	AO	Guidance	
8	**(a)**		Because if the collision where inelastic then the velocity of *A* after the collision would be 0 (ie *A* would 'stick to the wall') and so it would not collide with *B* again (which we know it does).	**B1** **[1]**	2.4		
	(b)		Recollide at same point => $v_{Ax} = 0$	**B1**	3.1b	Seen in solution	
			Com: $u\sin\theta = mv_B$	**M1**	3.3	or $v_{Ax} + mv_B$ on RHS	
			NEL: $e = \dfrac{v_B}{u\sin\theta}$	**M1**	3.3	or $e = \dfrac{v_B - v_{Ax}}{u\sin\theta}$	If v_{Ax} is used must be consistent in both equations
			=> $e = \dfrac{\left(\dfrac{u\sin\theta}{m}\right)}{u\sin\theta} = \dfrac{1}{m}$	**A1** **[4]**	1.1	AG ($v_B = eu\sin\theta$)	

	(c)		Velocity of A in y direction unchanged Either A's velocity multiplied by (–)e or B's velocity multiplied by (–)(5/9)e after collision with wall. $\frac{d}{u\cos\theta}+\frac{d}{eu\cos\theta}$ or $\frac{d}{eu\sin\theta}+\frac{d}{\frac{5}{9}e^2u\sin\theta}$ $\frac{d}{u\cos\theta}+\frac{d}{eu\cos\theta}=\frac{d}{eu\sin\theta}+\frac{d}{\frac{5}{9}e^2u\sin\theta}$ $5\tan\theta+\frac{5\tan\theta}{e}=\frac{5}{e}+\frac{9}{e^2}$ $\frac{25}{2}e^2+\frac{25}{2}e-5e-9=0$ $25e^2+15e-18=0$ $(5e+6)(5e-3)=0 \Rightarrow e=-\frac{6}{5}$ or $\frac{3}{5}$ $0\le e\le 1 \Rightarrow e=\frac{3}{5}\Rightarrow m=\frac{1}{e}=\frac{5}{3}$	**B1** **B1** **M1** **M1** **M1** **A1** **A1** **[7]**	3.4 3.4 3.1b 2.1 1.1 1.1 3.2a	$v_{Ay}=u\cos\theta$ Ignore inconsistency between velocity and speed here. Calculating time of travel for either A or B from point of collision to wall and back Equating expressions that are in terms of theta, e or m and a single velocity Reducing equation to 3 term quadratic in e or m Could be BC. –1.2 or 0.6 AG. Correctly rejecting negative root	FT if sin and cos confusion in part b Must be the correct component Could see eg $d-r$ $\frac{-15\pm\sqrt{2025}}{50}$
	(d)		In practice the plane cannot be smooth **or** the discs may not be the same size/radius **or** air resistance may be significant **or** NEL is only an approximation	**B1** **[1]**	3.5b	Any sensible limitation	

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